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NAVAL POSTGRADUATE SCHOOL
Monterey, California

POSITION DETERMINATION WITH
LORAN-C TRIPLETS AND THE HEWLETT-PACKARD
HP-41CV PROGRAMMABLE CALCULATOR

by

Rex H. Shudde

September 1982

Approved for public release; distribution unlimited.

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Superintendent

David A. Schrady
Provost

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This report presents an algorithm and HP-41CV programs for position determination with Loran-C chains. Additional computational routines include the ability to calibrate Loran station triplet data to a known benchmark and ITD's (Indicated Time Delay's), predict ITD's at given positions, compute the geodesic (similar to great circle) bearing and distance from a fix to the destination and to compute the geodesic bearing and distance from any one location to another. Utility routines allow the user to transfer station pair data between the HP-41CV and magnetic cards, magnetic tape and an extended

function/memory module.

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The programs in this report are for use within the Navy, and they are presented without representation or warranty of any kind.

CONTENTS

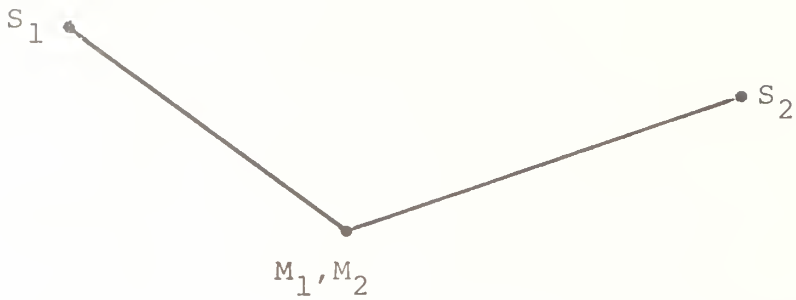
ABSTRACT

This report presents an algorithm and HP-41CV programs for position determination with Loran-C chains. Additional computational routines include the ability to calibrate Loran station triplet data to a known benchmark and ITD's (Indicated Time Delay's), predict ITD's at given positions, compute the geodesic (similiar to great circle) bearing and distance from a fix to the destination and to compute the geodesic bearing and distance from any one location to another. Utility routines allow the user to transfer station pair data between the HP-41CV and magnetic cards, magnetic tape and an extended function/ memory module.

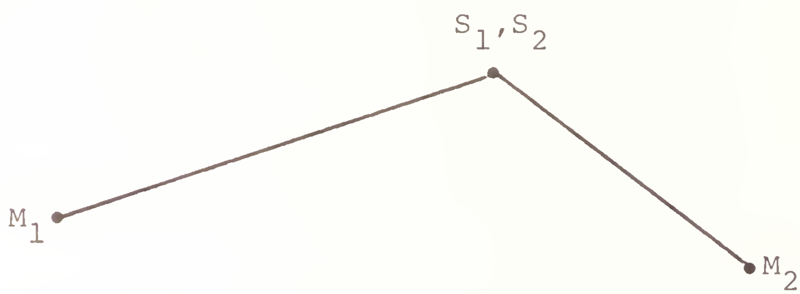
A. Introduction

The Loran system is a radio aid to navigation which utilizes the principle of hyperbolic fixing. The locus of points for which the difference in arrival time of synchronized signals from a pair of transmitters is constant determines a hyperbolic line of positions. The intersection of two hyperbolic lines of position from two pairs of stations determines position or a hyperbolic fix. That two pairs of stations are required for a fix does not necessarily mean that there are four separate stations, for one station of one pair may be colocated with one station of the other pair forming a Loran triplet (Figure 1). Triplets may be joined "end-to-end" by station colocation to form a Loran chain (Figure 2). Loran chains are common on both the East and West coasts of the North American continent.

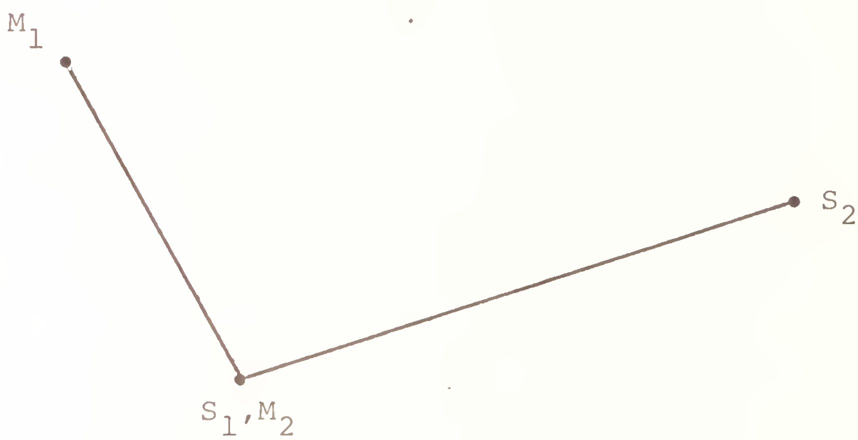
The present day Loran-C operates at 100-kHz and is in use in the Atlantic, Pacific and Mediterranean areas. The computational algorithm and programs described herein can be used for position determination with Loran-C triplets. Further information on the history, development and operation of the Loran systems may be found in References 1 and 2.



(a) Colocated Master Stations



(b) Colocated Slave Stations



(c) Colocated Master and Slave

Figure 1. Loran Triplets.

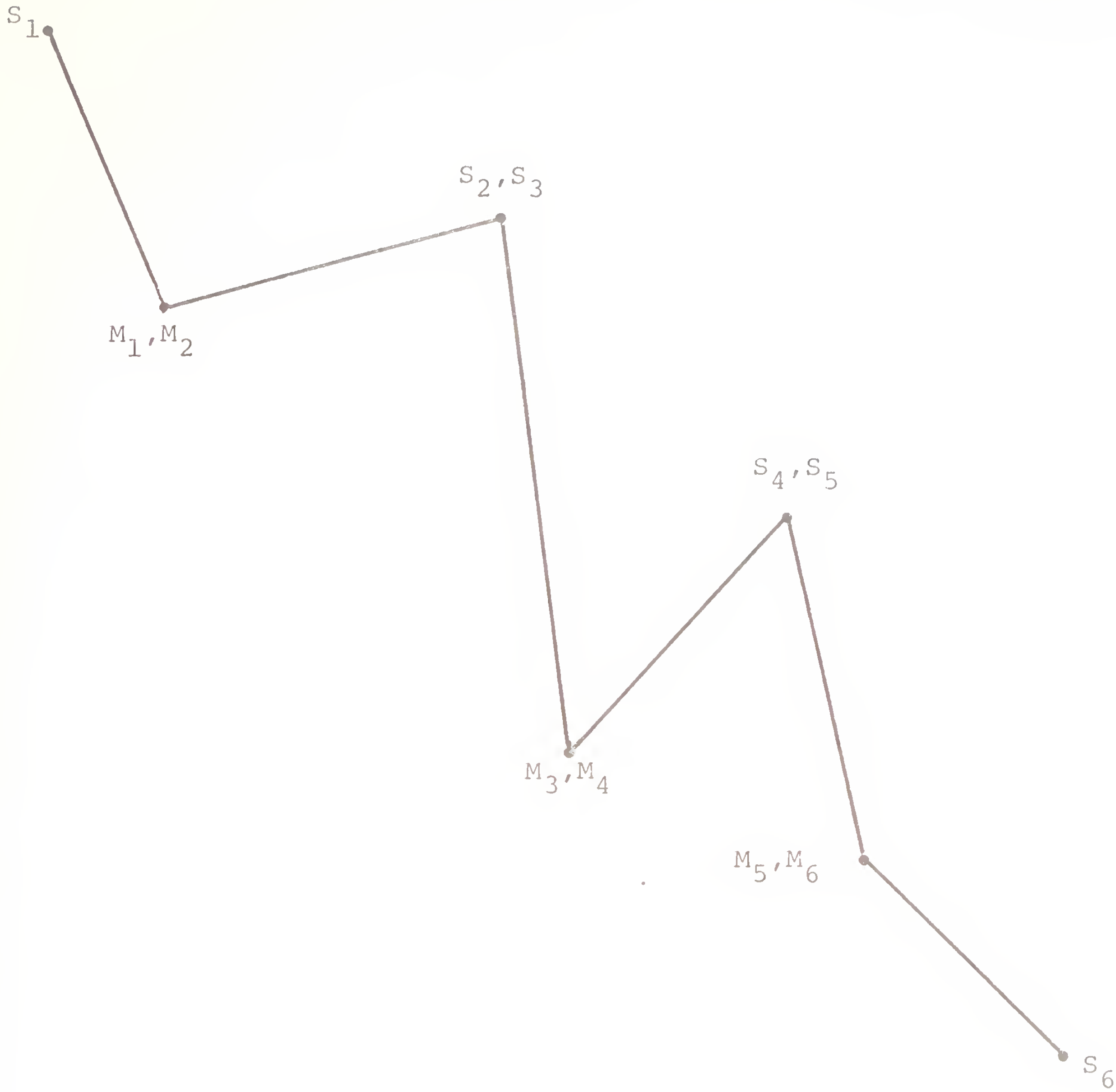


Figure 2. Loran Chain of Five Loran Triplets.

B. User Routines

1. Computational Routines:

FI - The FIxing routine is the main program for calculating a Loran-C fix from indicated time delays.

AS - The Alternate Solution routine will allow the second Loran fix solution to be computed. This routine toggles Flag 3 so that on subsequent fixes the FI routine will calculate the alternate solution.

DN - The DestiNation routine stores the latitude and longitude of a fixed destination.

HD - Computes the Heading and Distance from the current fix to the destination stored by the DN routine.

2. Manual Mode Routines:

MI - Manual Input allows station data to be input and stored via the calculator keyboard.

ED - Echo Data is a utility routine for validating station triplet information stored in the calculator.

3. Card Reader Routines:

CS - Card Store records station data onto magnetic cards.

CR - Card Read inputs station data from magnetic cards.

CE - Card Echo is a utility routine for validating station information stored on data cards.

4. Extended Memory Routines:

XS - XMEM Store records station data onto the extended memory module.

XR - XMEM Recall inputs station data from the extended memory module.

XD - XMEM Delete erases station data from the extended memory module.

5. Tape Cassette Routines:

TS - Tape Store records station data onto the tape cassette.

TR - Tape Recall inputs station data from the tape cassette.

TD - Tape Delete erases station data from the tape cassette.

6. Utility Routines:

DH - This routine is similiar to the HD option except that it computes the heading and distance from any origin to any destination.

PR - PR is used to PRedict the station ITD's that will be received at a given latitude and longitude.

CA - CA is the CAlibration option. Given the latitude and longitude of a known position and the indicated time delays from a Loran-C triplet, the stored station data are modified so that the FI routine (or AS) will compute the known position from the same time delays.

SW - SWitch data swaps the data of the two Loran stations stored in memory.

Note: There are no specific routines that relate to HP-41CV printer operations. However, all input and output will be recorded on a printer if one is attached.

C. Recommended Key Assignments

It is recommended that the following HP-41CV *user key* assignments be made and recorded onto the program cards if the program is first prepared manually:

			CA	ED
		CS	CR	CE
		TD	MI	SW
		TS	TR	DN
		XD	PR	DH
		XS	XR	HD
AS				
FI				

The *FI* function is placed on the ENTER key as a reminder that ENTER should *not* be used for data entry. To make the user key assignments on the HP-41CV, refer to the ASN function in the HP-41CV *Owner's Handbook and Programming Guide*.

D. <u>User Instructions with Examples</u>

1. <u>Manual Input Routine - MI</u>

This routine can be used to enter station data (Appendix C) to prepare data to be transferred to cards, extended memory, or tape cassette. It can also be used to enter station data manually if alternate data storage media are not available.

Example using 9940W and 9940Y. (Note: The notation <9>, for example, means that you must press the gold shift key and <u>then</u> the 9 key.)

	See	Key in	Press
1.		XEQ"MI"	
2.	ID?	<9><9><4><0>W	R/S
3.	CODE DELAY?	11000	R/S
4.	MS LAT?	39.330662	R/S
5.	MS LON?	118.495637	R/S
6.	SS LAT?	47.034799	R/S
7.	SS LON?	119.443953	R/S

If desired, these station parameters can now be transferred to card, extended memory or tape using the CS, XS, or TS routines, respectively. Otherwise, repeat the steps above with the 9940Y station data. These two stations will be used in the remaining examples.

	See	Key in	Press
1.			XEQ"MI"
2.	ID?	<9><9><4><0>Y	R/S
3.	CODE DELAY?	40000	R/S
4.	MS LAT?	39.330662	R/S

5. MS LON? 118.495637 R/S

6. SS LAT? 35.191818 R/S

7. SS LON? 114.481743 R/S

8. NEXT OPTION?

At this point the station data for 9940W and 9940Y are stored in the calculator. The 9940Y data can be transferred to card, extended memory or tape cassette using the CS, XS or TS routines, respectively.

[Advanced User Note: The first action taken in the MI routine is to XEQ"SW". SW is the data swap routine which exchanges to content of R16 - R25 with R26 - R35. The incoming data are then stored in the R16 - R25 registers. The CS, XS and TS routines transfer the content of R16 - R25 to card, extended memory or tape cassette, respectively. If desired, the content of these registers can be swapped once more by using the SW utility routine.]

2. Echo Data Routine - ED

This routine allows the user to review the station data resident in the calculator.

Example: Load the station pairs 9940W and 9940Y using either the MI, CR, XR or TR routines.

	See	Key in	Press
1.		XEQ"ED"	
2.	ID: 9940W		R/S
3.	CD: 11000		R/S
4.	MLT: 39.330662		R/S
5.	MLN: 118.495637		R/S
6.	SLT: 47.034799		R/S
7.	SLN: 119.443953		R/S
8.	BL: 2796.903		R/S
9.	ID: 9940Y		R/S
10.	CD: 40000		R/S
11.	MLT: 39.330662		R/S
12.	MLN: 118.495637		R/S
13.	SLT: 35.191818		R/S
14.	SLN: 114.481743		R/S
15.	BL: 1967.302		R/S
16.	NEXT OPTION?		

Notation: CD = coding delay, M = master, S = slave, LT = latitude, LN = longitude and BL is the station pair baseline plus the secondary phase correction in microseconds.

3a. Card Store Routine - CS

With the card reader attached, station data (in R16 - R25), which has been input using the MI, XR or TR routine, can be transferred to magnetic card using the XEQ"CS" command or by pressing the appropriate user defined key.

Example using 9940W.

	See	Key in	Press
1.		XEQ"PD"	
2.	WRITE: 9940W	(Pass a blank card through the card reader. Label the card track "9940W")	
3.	NEXT OPTION?		

To proceed with the remaining examples it is recommended that you also prepare a card for the 9940Y station pair.

3b. Card Read Routine - CR

With the card reader attached, XEQ"CR" or press the appropriate user defined key. This routine can be used to input the data for two station pairs, which must form a triplet.

Example using 9940W and 9940Y.

	See	Key in	Press
1.		XEQ"CR"	
2.	1ST CARD	(Pass the data card for 9940W through the card reader.)	
3.	2ND CARD	(Pass the data card for 99440Y through the card reader.)	
4.	NEXT OPTION?		

3c. Card Echo Routine - CE

With the card reader attached, XEQ"CE" or press the appropriate user defined key. This routine is used to validate the content of data cards against the table in Appendix C.

Example using 9940W.

	See	Key in	Press
1.		XEQ"CE"	
2.	STA. CARD	(Pass one side of a data card [9940W] through the card reader).	
3.	ID: 9940W		R/S
4.	CD: 11000		R/S
5.	MLT: 39.330662		R/S
6.	MLN: 118.495637		R/S
7.	SLT: 47.034799		R/S
8.	SLN: 119.443953		R/S
9.	BL: 2796.903		R/S
10.	NEXT OPTION?		

Notation: CD = coding delay, M = master, S = slave, LT = latitude, LN = longitude and BL is the station pair baseline plus the secondary phase correction in microseconds.

4a. Store Data in Extended Memory - XS

With the extended memory module in the HP-41CV, station data (in R16 - R25), which has been input using the MI, CR, XR or TR routine, can be transferred to the module using the XEQ"XS" command or by pressing the appropriate user defined key. Example using 994ØW.

	See	Key in	Press
1.		XEQ"XS"	
2.	NEXT OPTION?		

Should the station pair already be in extended memory, the message DUP FL (duplicate file) will be displayed. If needed, the duplicate file may be erased using the XD routine.

To proceed with the remaining examples it is recommended that you also store the 994ØY station pair.

4b. Recall Data from Extended Memory - XR

With the extended memory module installed, XEQ"XR" or press the appropriate user defined key to input the data for a station pair.

Example using 994ØW. (Note: The notation <9>, for example, means that you must press the gold shift key and then the 9 key.)

	See	Key in	Press
1.		XEQ"XR"	
2.	ID?	<9><9><4><Ø>W	R/S
3.	NEXT OPTION?		

4c. <u>Delete Data from Extended Memory - XD</u>

XEQ"XD" or press the appropriate user defined key to delete the specific station pair data from the extended memory module.

Example using 994ØW.

	See	Key in	Press
1.		XEQ"XD"	
2.	ID?	<9><9><4><Ø>W	R/S
3.	NEXT OPTION?		

The message FL NOT FOUND will be displayed if the file you wish to delete in not in the extended memory.

<u>Note:</u> The extended functions/ memory module will accommodate the data for 11 station pairs. The extended memory module will accommodate the data for an additional 22 station pairs.

5a. Store Data in the Tape Cassette - TS

With the tape cassette attached to the HP-41CV, station data (in R16 - R25), which has been input using the MI, CR, XR or TR routine, can be transferred to the tape using the XEQ"TS" command or by pressing the appropriate user defined key.

Example using 9940W.

	See	Key in	Press
1.		XEQ"TS"	
2.	NEXT OPTION?		

Should the station pair already be in extended memory, the message DUP FL NAME (duplicate file name) will be displayed. If needed, the duplicate file may be erased using the TD routine.

To proceed with the remaining examples it is recommended that you also store the 9940Y station pair.

5b. Recall Data from the Tape Cassette -TR

With the tape cassette attached, XEQ"TR" or press the appropriate user defined key to input the data for a station pair.

Example using 9940W. (Note: The notation <9>, for example, means that you must press the gold shift key and then the 9 key.)

	See	Key in	Press
1.		XEQ"TR"	
2.	ID?	<9><9><4><0>W	R/S
3.	NEXT OPTION?		

5c. <u>Delete Data from the Tape Cassette - TD</u>

XEQ"TD" or press the appropriate user defined key to delete the specific station pair data from the tape cassette.

Example using 9940W.

	See	Key in	Press
1.		XEQ"TD"	
2.	ID?	<9><9><4><0>W	R/S
3.	NEXT OPTION?		

The message FL NOT FOUND will be displayed if the file to be deleted is not on the tape.

6. Loran-C Fixing Routines FI and AS

Given the indicated time delay (ITD) from two station pairs which form a triplet, a Loran-C fix is obtained.

Example: Load 9940W and 9940Y into the calculator using the MI, CR, XR or TR routine. The ITD on 9940W is 16019 microseconds and the ITD of 9940Y is 42585 microseconds. Where are you?

See	Key in	Press
1.	XEQ"FI"	
2. ITD: 9940W	16019	R/S
3. ITD: 9940Y	42585	R/S
4. LAT: 39.1419		R/S
5. LON: 115.5052		R/S
6. NEXT OPTION?		

Since you are on a boat, you know that you cannot be in central Nevada at 39d14'19" North and 115d50'52" West. Every Loran-C fix has two solutions, so in this case you must use the alternate solution.

See	Key in	Press
7.	XEQ"AS"	
8. LAT: 35.0001		R/S
9. LON: 125.0009		R/S
10. NEXT OPTION?		

This is the proper solution at almost exactly 35 degrees North and 125 degrees West. Note that annunciator 3 (Flag 3) shows in the display indicating the alternate solution. If you should now repeat from Step 1, you will obtain the proper solution immediately.

The message "E: NO TRIPLET" will appear following Step 1 if the data do not comprise a valid triplet. The latitudes and longitudes of each station pair at the vertex must agree exactly. Should this error occur, use the ED routine to review the resident station data.

The message "E: ITD ERROR" will appear following Step 2 or 3 indicating that the ITD you keyed in is inconsistent with the station parameters. Press R/S to be requeried for the ITD.

7a. Distance and Heading Routines DN and HD

If you know the latitude and longitude of your destination, you may key these in and then see how far your fix is from your destination and what the geodesic heading (similiar to great circle heading) is to your destination.

Example: Your destination is Moss Landing at about 36d48'N and 121d47'W. Your current fix is 35dN and 125dW (see the FI-SA example). First, key in your destination.

See	Key in	Press
1.	XEQ"DN"	
2. DEST LAT?	36.48	R/S
3. DEST LON?	121.47	R/S
4. NEXT OPTION?		R/S

The destination is now stored in the calculator and will remain unchanged until you use either the DN or DH options. Also, the latest fix is stored and will remain unchanged until you use either the FI, AS or DH options. Now, find the distance and bearing from the latest fix (see the FI-AS example) to Moss Landing.

See	Key in	Press
1.	XEQ"HD"	
2. N.MI: 190.38		R/S
3. BRG: 54.3411		R/S
4. NEXT OPTION?		R/S

So the distance to Moss Landing is 190.38 nautical miles at a heading of 54d34'11".

7b. Distance and Heading Routine - DH

Given the latitude and longitude of an origin and destination, this routine will find the distance and heading from one to the other.

Example: How far, and in what direction, is Corvallis, Oregon (44d34'N, 123d16'W) from Cupertino, California (37d19N, 122dØ2'W)?

	See	Key in	Press
1.		XEQ"DH"	
2.	ORIG LAT?	37.19	R/S
3.	ORIG LON?	122.Ø2	R/S
4.	DEST LAT?	44.34	R/S
5.	DEST LON?	123.16	R/S
6.	N.MI: 438.32		R/S
7.	BRG: 353.Ø259		R/S
8.	NEXT OPTION?		

Thus the distance is 438.32 nautical miles and the direction of Corvallis from Cupertino is 353dØ2'59".

8. <u>ITD Prediction Routine - PR</u>

As an aid to identification, this routine will allow the user to determine what ITD's should be received at a given location.

Example: Suppose that you know you are somewhere near latitude 35 North and longitude 125 West but are not sure what ITD's you should be receiving from 9940W and 9940Y. To determine these ITD's, proceed as follows:

	See	Key in	Press
1.		XEQ"PR"	
2.	LAT?	35	R/S
3.	LON?	125	R/S
4.	9940W: 16019.35		R/S
5.	9940Y: 42584.71		R/S
6.	NEXT OPTION?		

You should expect to receive an ITD of 16019.35 from 9940W and an ITD of 42584.71 from 9940Y.

9. Calibration Routine - CA

This routine will allow the user to calibrate the Loran data in the calculator to a known position when the indicated time delay (ITD) is known for each station pair.

Example: Suppose you are receiving an ITD of 16308 from 9940W and 42800 from 9940Y. These ITD's would tell you that your location is 36d47'55"N and 121d47'11"W. However, you know that your position is bench marked to be at 36d47'36"N and 121d46'58"W, and you wish to calibrate your calculator so that the ITD's of 16308 and 42800 will give you the latter fix instead of the former. Proceed as follows:

See	Key in	Press
1.	XEQ"CA"	
2. LAT?	36.4736	R/S
3. LON?	121.4658	R/S
4. ITD 9940W:	16308	R/S
5. ITD 9940Y:	42800	R/S
6. NEXT OPTION?		

Entering 16308 and 42800 into the FI routine will now give you a fix at 36d47'35"N and 121d46'59"W. The small discrepancy between this fix and the bench mark is due to assumptions made in the fixing algorithm.

Calibration is achieved by modifying the Master/Slave baseline (BL in the CE and ED routines). See Section F.

10. Switch Data Registers Routine - SW

The SW utility allows the user to swap the station data stored in R16 - R25 with the data stored in R26 - R35. Whenever the MI, XR or TR routine is used, the SW routine is invoked prior to the loading of the data; the incoming data are then placed in R16 - R25. The consequence is that the first station pair data reside in R26 - R35 and the second station pair data reside in R16 - R25.

One user application of SW would be to change the order of the station ID query in the FI routine (this also affects the order of determination of the solution and alternate solution). Another user application would be to output both resident station pairs to card, extended memory or tape cassette using the CS, XS or TS routines, respectively. To accomplish this, first use the CS, XS, or TS routine; then XEQ"SW" (note that the ID of the station data in R16 - R25 appears in the display instead of the NEXT OPTION? prompt); and finally use the CS, XS or TS routine once more.

11a. Recording the Loran-C Program onto Magnetic Cards

(1) Attach the card reader to the HP-41CV.

(2) Place the calculator in the USER mode.

(3) Press the PRGM key to place the calculator in the program mode.

(4) Pass one side of a blank magnetic card through the card reader. Then follow the display prompts until nine program cards (17 tracks) have been recorded.

(5) Press the PRGM key once more to leave the program mode.

(6) To record a status card, XEQ"WSTS". Then pass a blank card through the card reader following the display prompts.

11b. Recording the Loran-C Program onto Magnetic Tape

(1) Attach the tape cassette to the HP-41CV.

(2) Place the calculator in the USER mode.

(3) Press the alpha key, key in the word LORANC, press the alpha key once more, and then XEQ"WRTP".

(4) To record the program status, press the alpha key, key in the word STATUS, press the alpha key once more, and then XEQ"WRTS".

12a. Loading the Loran-C Program from Magnetic Cards

(1) Attach the card reader to the HP-41CV.

(2) Clear program memory: Turn the calculator off, then, while pressing the left arrow (erase) key down, turn the calculator on.

(3) Place the calculator in the USER mode.

(4) Read in the STATUS card. The status card will set the calculator to SIZE 42.

(5) Read in the nine program cards (17 tracks).

12b. Loading the Loran-C program from Tape Cassette

(1) Attach the tape cassette to the HP-41CV.

(2) Clear program memory. (See Step 2 above).

(3) Place the calculator in the USER mode.

(4) Press the alpha key, key in the word STATUS, press the alpha key once more, and then XEQ"READS". The status file will set the calculator to SIZE 42.

(5) Press the alpha key, key in the word LORANC, press the alpha key once more, and then XEQ"READP".

E. The Loran-C Fixing Algorithm

The principles of Loran lines of position (LOP's) and fixing are adequately covered in Reference 1 and will not be repeated here.

The basic Loran-C equation [Ref. 4] can be written as

$$ITD = [T_S + p(T_S)] - [T_M + p(T_M)] + [T_B + p(T_B)] + \delta \quad (1)$$

where

- ITD is the "indicated time difference" in microseconds,
- T_M is the distance, in microseconds, from the master to the receiver,
- T_S is the distance, in microseconds, from the slave to the receiver,
- T_B is the distance, in microseconds, between the master and the slave,
- δ is the assigned station pair coding delay, in microseconds, and
- $p(T)$ is the secondary phase correction, in microseconds, for a surface seawater path of length T .

The quantity

$$\Delta t = T_B + p(T_B) + \delta$$

is a constant for each master/slave pair. The quantity T_B is computed from the positions of the master and slave using the reverse solution algorithm (Section H) at the time of manual data input (Routine MI).

The following World Geodetic System 1972 (WGS 72) values have been adopted for Loran-C navigation [Ref. 4]:

v_0 = 299792458 meters/second is the velocity of light in free space,

η = 1.000338 is the index of refraction of the surface of the earth for standard atmosphere and 100 kHz electromagnetic waves,

a_e = 6378135.000 meters is the equatorial radius of the earth

and f = 1/298.26 is the flattening factor ($1-b/a_e$, where b is the polar radius) of the earth.

Accurate formulas for computing the secondary phase correction p(T) are contained in Reference 4, but for use in the HP-41CV, the form

$$p(T) = a_0/T + a_1 + a_2T \qquad (2)$$

is used, where T is in microseconds and

1. For $T \geq 537$ μsec:

   $a_0 = 129$,

   $a_1 = -0.408$, and

   $a_2 = 0.0006458$.

2. For $T < 537$ μsec:

   $a_0 = 2.74$,

   $a_1 = -0.011$, and

   $a_2 = 0.00033$.

On the surface of a sphere, a hyperbolic line of position (LOP) can be represented by the equation [Ref. 1, page 175]

$$\tan r = \frac{\cos 2a - \cos 2c}{\sin 2c \cos \omega + \zeta \sin 2a} \tag{3}$$

where the origin of the coordinate system is at the prime focus of the spherical hyperbola, 2c is the spherical arc joining the foci, 2a is a constant for any one hyperbola, and r and ω are the spherical coordinates of a point on the hyperbola. If the base line of the coordinate system is the arc joining the foci the ω is the spherical polar angle from the baseline to a point P on the spherical hyperbola and r is spherical polar distance (or arc) from the prime focus to P. Using the Loran system we take $\zeta = +1$ if the prime focus is at a master station and $\zeta = -1$ if the prime focus is at a slave station.

If we let $v = v_0/\eta$ be the velocity of 100kHz electromagnetic radiation at the earth's surface then, for a spherical earth, we can relate the parameters in Equations 1 and 3 as follows:

$$2c = vT_B/a_e ,$$

and

$$2a = v(T_S - T_M)/a_e .$$

Using the spherical approximation for now, we see that 2c is known for any Loran pair. The "indicated time delay" ITD is measured by the receiver at point P, and to determine a hyperbolic line of position we must determine 2a, but $T_S - T_M$ cannot be computed from Equations 1 and 2. If a_0 were zero in Equation 2, then it would be possible to determine $T_S - T_M$ uniquely. As an approximation we use the following parameters in Equation 2:

$$a_0 = 0 ,$$

$$a_1 = -0.321 ,$$

and $$a_2 = 0.000635 .$$

These values have been obtained by setting $a_0 = 0$ and determining a_1 and a_2 by linear regression of the $T > 537$ values over the interval of $1000 < T < 8000$. This approximation is quite good (within 0.03 μs) for distances up to 10,000 microseconds where small changes in the LOP's can cause large position errors. At short distances the error increases from 0.05 μs at 1000 μs to 0.58 μs at 10 μs; although these errors are large for small distances, the LOP's are not as sensitive to these changes as they would be at large distances. When this approximation is substituted into Equation 1, we obtain

$$[T_S + a_1 + a_2 T_S] - [T_M + a_1 + a_2 T_M] = ITD - \Delta t ,$$

or

$$T_S - T_M = (ITD - \Delta t)/(1 + a_2) \tag{4}$$

and hence $2a = v(T_S - T_M)/a_e$ is determined for use in the spherical approximation.

Consider a Loran-C triplet with the master stations co-located. Let ξ_1 and ξ_2 denote the azimuth angles of slave 1 (S_1) and slave 2 (S_2), respectively, measured from North toward the East from the master stations (M) (see Figure 3).

Further, let α and r be the azimuth and spherical polar arc (distance) of the receiver (R) from M. For this geometry, Equation 3 can be written in the form

$$\tan r_i = \frac{B_i}{C_i \cos(\alpha - \xi_i) + A_i} , \tag{5}$$

where

$$A_i = \zeta_i \sin 2a_i ,$$

$$B_i = \cos 2a_i - \cos 2c_i ,$$

and

$$C_i = \sin 2c_i$$

for the i^{th} Loran pair, $i = 1,2$. Since $r_1 = r_2 = r$, we can eliminate $\tan r$ between the two equations. The resulting equation can be rewritten as

$$C \cos \alpha + S \sin \alpha = K , \tag{6}$$

where

$$C = B_1C_2 \cos \xi_2 - B_2C_1 \cos \xi_1 ,$$

$$S = B_1C_2 \sin \xi_2 - B_2C_1 \sin \xi_1 ,$$

and

$$K = B_2A_1 - B_1A_2 .$$

If we define $\rho > 0$ and γ by the equations

$$\rho \cos \gamma = C , \tag{7}$$

and

$$\rho \sin \gamma = S ,$$

then

$$\rho = \sqrt{C^2 + S^2} ,$$

and

$$\gamma = \text{qatn } (S,C) .$$

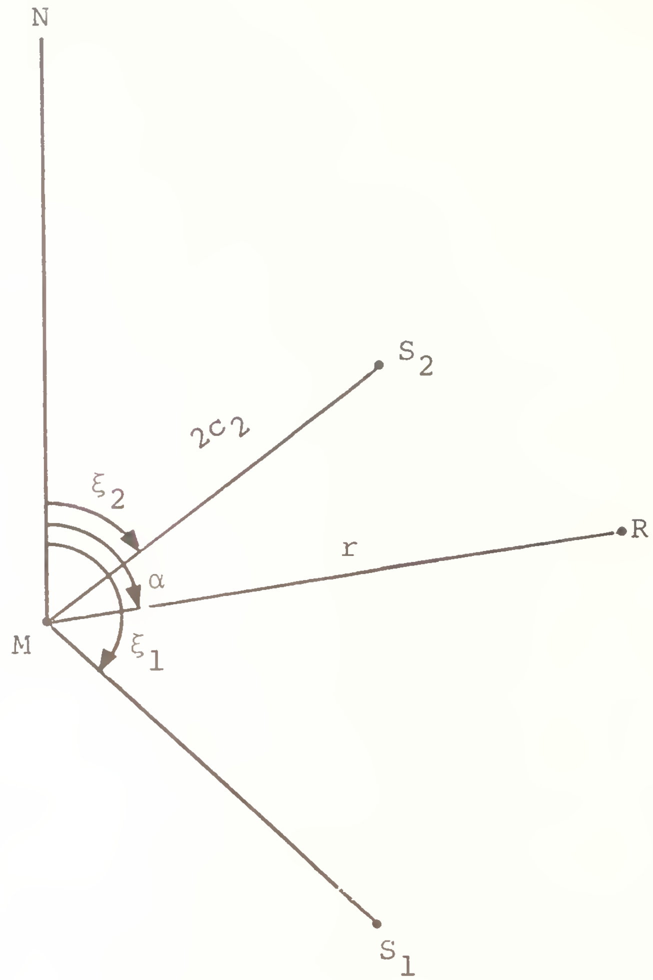


Figure 3. Geometry of a Loran Triplet and a Receiver.

Here the function qatn(y,x) is the arctangent of y/x adjusted for the proper quadrant according to the signs of x and y. A compact form of this function is

$$\text{qatn}(y,x) = \tan^{-1} \frac{y}{x + 10^{-9}\, t(x = 0?)} + \pi t(x < 0?)$$

where $t(z) = 1$ when z is true

and $t(z) = 0$ when z is false.

When convenient we will use the notation qatn(y/x) interchangeably with qatn(y,x). Now we can substitute Eq. (7) into Eq. (6) and solve for

$$\alpha = \gamma \pm \cos^{-1}(K/\rho) \qquad (8)$$

to obtain the azimuth angle α of the two points of intersection of the spherical hyperbolic LOP's. Finally we can obtain a value for r by substituting each α into Eq. (5) for either $i = 1$ or $i = 2$. We find that

$$r = \text{qatn}\left[\frac{B_i}{C_i \cos(\alpha - \xi_i) + A_i}\right] \text{ for } i = 1 \text{ or } 2. \qquad (9)$$

The distance and azimuth from M or the triplet vertex can be converted into the latitude and longitude of the two possible positions of r.

The fixing algorithm then uses α and r in the direct solution algorithm of spheroidal geodesy (Section G).

F. The Calibration and ITD Prediction Algorithms

Calibration can be achieved when an ITD is measured at a known bench marked position. From the bench marked position and the known master and slave positions, the quantities $T_M + p(T_M)$ and $T_s + p(T_s)$ can be computed using the reverse solution algorithm (Section H) and the accurate secondary phase correction formula (Eq. 2). Equation 1 can then be solved for $T_B + p(T_B)$ to obtain a modified baseline. This modified baseline is stored and then used instead of the true baseline in subsequent computations. The affect on the accuracy of fixes using this modified baseline with positions far removed from the bench mark has not studied.

The ITD prediction algorithm is a direct application of Equation 1. A known position, together with the known master and slave positions, is used to compute the quantities $T_M + p(T_M)$ and $T_s + p(T_s)$. When these values, along with a computed or calibrated baseline, $T_B + p(T_B)$, are substituted into Equation 1, a predicted ITD is obtained.

G. The Direct Solution Algorithm

This direct solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas [Ref. 5, pp. 7-8]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in a previous section. East longitudes and South latitudes are negative. We are given the point $P_1(\phi_1, \lambda_1)$ on the spheroid, where ϕ_1, λ_1 are the geodetic latitude and longitude (geographic coordinates); the forward azimuth α_{12} and distance S to a second point $P_2(\phi_2, \lambda_2)$; and from these we are to find the geographic coordinates ϕ_2, λ_2 and the back azimuth α_{21}. The given quantities are ϕ_1, λ_1, α_{12} and S. No assumptions about the relative location of P_1 and P_2 are required. The modified direct solution algorithm is:

$$\theta_1 = \tan^{-1}[(1-f)\tan\theta_1],\ M = -\sin\alpha_{12}\cos\theta_1\ ,$$

$$C_1 = fM,\ C_2 = f(1-M^2)/4\ ,$$

$$D = (1-C_2)(1-C_2 - C_1 M),\ P = C_2[1 + (1/2)C_1 M]/D\ ,$$

$$N = \cos\theta_1 \cos\alpha_{12}\ ,\ \sigma_1 = \mathrm{qatn}(N, \sin\theta_1)\ ,$$

$$d = S/(a_e D)\ ,\ u = 2(\sigma_1 - d)\ ,\ W = 1-2P\cos u\ ,$$

$$V = \cos(u+d)\ ,\ X = C_2^2 \sin d \cos d(2V^2-1)\ ,$$

$$Y = 2PVW\sin d\ ,\ \Delta\sigma = d + X - Y\ ,$$

$$K = [M^2 + (N\cos{-\Delta\sigma}\sin\theta_1 \sin\Delta\sigma)^2]^{1/2}\ ,$$

$$\theta_2 = \tan^{-1}[(\sin\theta_1\cos\Delta\sigma + N\sin\Delta\sigma)/K]\ ,$$

$$\Delta\eta = \mathrm{qatn}(-\sin\Delta\sigma\sin\alpha_{12}\ ,\ \cos\theta_1\cos\Delta\sigma - \sin\theta_1\sin\Delta\sigma\cos\alpha_{12})\ ,$$

$$H = C_1(1-C_2)\ \Delta\sigma - C_1 C_2 \sin\Delta\sigma\cos(2\sigma_1 - \Delta\sigma)\ ,$$

$$\lambda_2 = \lambda_1 + \Delta\eta - H\ ,$$

$$\alpha_{21} = \mathrm{qatn}[-M, -(N \cos \Delta\sigma - \sin \theta_1 \sin \Delta\sigma)] ,$$

$$\phi_2 = \tan^{-1}[\tan \theta_2/(1-f)]$$

Details of the modifications made to Thomas' algorithm are contained in Reference 3. The algorithm above has been further modified so that Eastern longitudes, rather than Western longitudes, are negative.

H. The Reverse Solution Algorithm

This reverse solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas [Ref. 5, pp. 8-10]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in a previous section. East longitudes (λ) and South latitudes (ϕ) are negative. We are given the points $P_1(\phi_1,\lambda_1)$, $P_2(\phi_2,\lambda_2)$ on the spheroid and are to find the distance S between the points and the forward and back azimuths, α_{12} and α_{21}. Given quantities are ϕ_1, λ_1, ϕ_2 and λ_2. No assumptions about the relative location of P_1 and P_2 are required. The modified reverse solution algorithm is:

$$\theta_i = \tan^{-1}[(1-f)\tan\phi_i] \ , \ i = 1,2 \ ,$$

$$\Delta\lambda = \lambda_2 - \lambda_1 \ , \ \Delta\theta_m = (\theta_2 - \theta_1)/2 \ , \ \theta_m = (\theta_1 + \theta_2)/2 \ ,$$

$$H = \cos^2\Delta\theta_m - \sin^2\theta_m \ , \ L = \sin^2\Delta\theta_m + H\sin^2(\Delta\lambda/2) \ ,$$

$$d = 2\sin^{-1}\sqrt{L} \ , \ U = 2\sin^2\theta_m\cos^2\Delta\theta_m/(1-L) \ ,$$

$$V = 2\sin^2\Delta\theta_m\cos^2\theta_m/L \ , \ X = U + V \ , \ Y = U - V \ ,$$

$$T = d/\sin d \ , \ D = 4T^2 \ , \ E = 2\cos d \ , \ A = DE \ ,$$

$$C = T - (A-E)/2, \ n_1 = X(A+CX) \ ,$$

$$B = 2D, \ n_2 = Y(B+EY) \ , \ n_3 = DXY \ ,$$

$$\delta_2 d = f^2(n_1 - n_2 + n_3)/64 \ , \ \delta_1 d = f(TX-Y)/4 \ ,$$

$$S/a_e = (T - \delta_1 d + \delta_2 d)\sin d \ , \ M = 32T - (20T-A)X - (B+4)Y \ ,$$

$$F = 2Y - E(4-X) \ , \ G = fT/2 + f^2 M/64 \ , \ Q = -(FG\tan\Delta\lambda)/4 \ ,$$

$$\Delta\lambda_m' = (\Delta\lambda + Q)/2 \ ,$$

$$t_1 = \text{qatn}(\sin \Delta\theta_m \cos \Delta\lambda'_m \, , \cos \theta_m \sin \Delta\lambda'_m) \, ,$$

$$t_2 = \text{qatn}(- \cos \Delta\theta_m \cos \Delta\lambda'_m \, , \sin \theta_m \sin \Delta\lambda'_m) \, ,$$

$$\alpha_{12} = t_1 + t_2 \, , \; \alpha_{21} = t_1 - t_2$$

Details of the modifications made to Thomas' algorithm are contained in Reference 3. The algorithm above has been further modified so that Eastern longitudes, rather than Western longitudes, are negative.

I. Program Accuracy

The direct and reverse solution algorithms are equivalent to the second order flattening algorithms given by Thomas (Ref. 5); the parameters of the WGS 1972 spheroid are used. The reverse solution algorithm reproduces the baselines provided by the Defense Mapping Agency for all 40 Loran-C stations to within 0.15 meters (the average deviation is -0.031 meters, DMA minus HP-41CV, with a standard deviation of 0.037 meters) and to within 0.01 microseconds, including the secondary phase correction for an all seawater path. The reverse solution algorithm is also used to generate predicted ITD's; these are presumed to be within 0.01 microseconds, also.

The fixing algorithm uses the direct solution algorithm with the azimuth and distance of the fix from the vertex of the Loran-C triplet computed from Equations 8 and 9 as inputs. Equations 8 and 9 are based upon spherical geometry and include an approximation to the secondary phase correction for an all seawater path. The largest source of error is the assumption that the azimuth and distance to the fix are accurately represented by this spherical approximation. This approximation has not been rigorously tested, however it is possible to use the reverse solution algorithm to predict the ITD's that will be received at a given position and then to enter these ITD's into the fixing algorithm to determine how accurately the fixing algorithm reproduces the original position. The distance between the fix and the original position can be determined using the HD algorithm. Tables 1, 2 and 3 were producted in this manner. Similiar tables with different station pairs are given in References 3, 6 and 7. It is felt that the results are accurate enough to not warrant the inclusion of an iterative improvement routine. Of the samples in Tables 1, 2 and 3, the largest error, Ø.22 n.mi., is the first entry in Table 1. From the chart LCNC-2, it estimated that the angle of intersection of the two hyperbolic lines of position is about 5 degrees and so an error of Ø.22 n.mi. should not be unexpected.

Table 1. Colocated Master Stations

Position		Predicted ITD's		HP-41CV Fix		
Lat	Long	9940W	9940X	Lat(N)	Long(W)	Error n.mi
31°	123°	16413.28	27570.93	30°59'47"	123°00'03"	0.22
37°	126°	15610.11	27020.50	36°59'53"	126°00'09"	0.17
42°	129°	13881.78	27285.58	42°00'00"	129°00'00"	0
44°	132°	13180.89	27371.19	44°00'00"	132°00'01"	0.01
48°	135°	12301.25	27552.06	48°00'01"	135°00'03"	0.03
50°	138°	12068.67	27584.22	50°00'01"	138°00'04"	0.05

Table 2. Colocated Slave Stations

Position		Predicted ITD's		HP-41CV Fix		
Lat	Long	9940W	5990Y	Lat(N)	Long(W)	Error n.mi
31°	123°	16413.28	27177.18	31°00'03"	122°59'58"	0.06
37°	126°	15610.11	27403.20	37°00'01"	125°59'59"	0.01
42°	129°	13881.78	27955.45	42°00'00"	129°00'00"	0
44°	132°	13180.89	28512.90	44°00'00"	132°00'00"	0
48°	135°	12301.25	29413.61	48°00'00"	134°59'59"	0.01
50°	138°	12068.67	29816.84	50°00'00"	137°59'59"	0.02

Table 3. Colocated Master and Slave Stations

Position		Predicted ITD's		HP-41CV Fix		
Lat	Long	5930Y	9960W	Lat(N)	Long(W)	Error n.mi
44°	63°	29864.46	11685.15	44°00'00"	63°00'00"	0
41°	66°	30585.61	12946.91	41°00'00"	66°00'00"	0
39°	69°	31020.46	14111.31	39°00'00"	69°00'00"	0
35°	72°	31064.57	15139.48	35°00'00"	72°00'00"	0
30°	75°	31040.82	15610.46	29°59'59"	75°00'01"	0.02
26°	78°	31106.20	15858.46	25°59'57"	78°00'02"	0.05

J. References

1. J. A. Pierce, A. A. McKenzie and R. H. Woodward, editors, LORAN, M.I.T. Radiation Laboratory Series, McGraw-Hill Book Company, Inc., 1948.

2. G. Hefley, The Development of Loran-C Navigation and Timing, National Bureau of Standards Monograph 129, U.S. Department of Commerce, U.S. Government Printing Office, Washington, D.C. 20402, October 1972.

3. R. H. Shudde, "An Algorithm for Position Determination Using Loran-C Triplets with a BASIC Program for the Commodore 2001 Microcomputer", Technical Report NPS55-80-009, March 1980, Naval Postgraduate School, Monterey, CA 93940.

4. General Specifications for Loran-C (20 June 1977) and Loran-C Constants were obtained from the Chief, Navigation Department, Defense Mapping Agency, Hydrographic/Topographic Center, Washington, D.C. 20315.

5. Paul D. Thomas, "Spheroidal Geodesics, Reference Systems, and Local Geometry", PS-138, U.S. Naval Oceanographic Office, Washington, D.C., January 1970.

6. R. H. Shudde, "Position Determination with LORAN-C Triplets and the Hewlett-Packard HP-67/97 Programmable Calculators", Technical Report NPS55-80-010, March 1980, Naval Postgraduate School, Monterey, CA 93940.

7. R. H. Shudde, "Position Determination with LORAN-C Triplets and the Texas Instruments TI-59 Programmable Calculator", Technical Report NPS55-80-020, May 1980, Naval Postgraduate School, Monterey, CA 93940.

Appendix A: Program Storage Allocations, Flag Usage and Program Listing

Program Size: 42

Registers:

R00 - R13: Scratch storage

R14: Flattening

2nd Sta. Pair	Variable	1st Sta. Pair
R15:	2a	
R16:	ID	:R26
R17:	$T_B + p(T_B)$	:R27
R18:	2c	:R28
R19:	CD	:R29
R20:	θ master	:R30
R21:	λ master	:R31
R22:	α master-slave	:R32
R23:	θ slave	:R33
R24:	λ slave	:R34
R25:	α slave-master	:R35
	2a	:R36
R37:	θ fix	
R38:	λ fix	
R39:	α fix-dest	
R40:	θ dest	
R41:	λ dest	

Flag Usage:

F00: 1. Requery erroneous ITD at LBL06

2. Echo two data sets at LBL ED

F01: PR and CA interlock

F03: Set for 2nd solution

F05: Set if slave at vertex of 2nd station pair

F06: Set if slave at vertex of 1st station pair

F07: Set for XMEM functions, clear for tape functions

F08: DN and DH interlock

F09: Set if DMS conversion required

F10: 1. SW and AS interlock

2. Input interlock in FI vertex check

F14: PR and CA loop control

01•LBL "LOR ANC"	DIRECT SOLUTION
02•LBL 90	θ_1, λ_1, α_{12} and S/a_e are in the T, Z, Y and X registers
03 STO 00	
04 CLX	
05 1	
06 P-R	
07 STO 01	cos α_{12}
08 RDN	
09 CHS	
10 STO 02	- sin α_{12}
11 RDN	
12 STO 03	λ_1
13 CLX	
14 1	
15 P-R	
16 STO 04	cos θ_1
17 RCL 02	
18 *	
19 STO 06	M
20 X<>Y	
21 STO 05	sin θ_1
22 X<>Y	
23 RCL 14	f
24 *	
25 STO 07	c_1
26 1	
27 RCL 06	
28 X↑2	
29 -	
30 4	
31 /	
32 RCL 14	
33 *	
34 STO 08	c_2
35 1	
36 -	
37 CHS	

38 ENTER↑	
39 ENTER↑	
40 RCL 07	
41 RCL 06	
42 *	
43 -	
44 *	
45 STO 09	D
46 1/X	
47 RCL 08	
48 *	
49 1	
50 RCL 07	
51 RCL 06	
52 *	
53 2	
54 /	
55 +	
56 *	
57 STO 10	P
58 RCL 04	
59 RCL 01	
60 *	
61 STO 11	N
62 RCL 05	
63 RAD	
64 R-P	
65 X<>Y	
66 STO 12	σ_1
67 RCL 00	
68 RCL 09	
69 /	
70 STO 09	
71 RCL 12	
72 -	
73 CHS	
74 ENTER↑	
75 +	

Program Listing

76 STO 13	u	114 RCL 09	
77 COS		115 +	
78 RCL 10		116 STO 13	
79 *		117 1	
80 ENTER↑		118 P-R	
81 +		119 RCL 11	
82 1		120 *	N cos Δσ
83 -		121 X<>Y	
84 CHS	w	122 RCL 05	
85 RCL 13		123 *	
86 RCL 09		124 -	
87 +		125 STO 10	N cos Δσ - sin θ_1 sin Δσ
88 COS		126 X↑2	
89 STO 13	V	127 RCL 06	
90 X↑2		128 X↑2	M^2
91 ENTER↑		129 +	
92 +		130 SQRT	K
93 1		131 1/X	
94 -		132 RCL 13	
95 RCL 09	d	133 1	
96 1		134 P-R	
97 P-R		135 RCL 05	
98 *		136 *	
99 *		137 X<>Y	
100 RCL 08		138 RCL 11	
101 X↑2		139 *	
102 *	X	140 +	
103 X<>Y		141 *	
104 RCL 13	V	142 ATAN	
105 *		143 R-D	
106 RCL 10	P	144 STO 09	θ_2
107 *		145 RCL 13	
108 RCL 09	d	146 SIN	
109 SIN		147 RCL 02	
110 *		148 *	
111 ENTER↑		149 RCL 13	
112 +	Y	150 1	
113 -		151 P-R	

152 RCL 04		190 R-P	
153 *		191 CLX	
154 X<>Y		192 RCL 06	
155 RCL 05		193 CHS	
156 *		194 RCL 10	
157 RCL 01		195 CHS	
158 *		196 R-P	
159 -		197 RDN	
160 R-P	$\Delta\eta$	198 X<>Y	α_{21}, λ_2 and θ_2 are in the
161 CLX		199 RCL 09	Z, Y and X registers
162 1		200 RTN	
163 RCL 08		201♦LBL 80	REVERSE SOLUTION
164 -		202 X<>Y	
165 RCL 07		203 RDN	Stack contains θ_1, λ_1, θ_2
166 *		204 -	and λ_2 in the T,Z,Y and
167 RCL 13		205 CHS	
168 *		206 STO 00	$\Delta\lambda$ X registers
169 RCL 12		207 RDN	
170 ENTER↑		208 -	
171 +		209 LASTX	
172 RCL 13		210 X<>Y	
173 -		211 2	
174 COS		212 /	$\Delta\theta_m$
175 RCL 13		213 ENTER↑	
176 SIN		214 ENTER↑	
177 *		215 1	
178 RCL 07		216 P-R	
179 *		217 STO 01	$\cos \Delta\theta_m$
180 RCL 08		218 X↑2	
181 *		219 RDN	
182 -	H	220 STO 02	$\sin \Delta\theta_m$
183 -		221 RDN	
184 R-D		222 +	
185 RCL 03		223 1	
186 +		224 P-R	
187 DEG	λ_2	225 STO 03	$\cos \theta_m$
188 1		226 RDN	
189 P-R		227 STO 04	$\sin \theta_m$

```
228 X↑2
229 -           H
230 RCL 00
231 2
232 /
233 SIN
234 X↑2
235 *
236 RCL 02
237 X↑2
238 +
239 STO 05      L
240 SQRT
241 RAD
242 ASIN
243 ENTER↑
244 +
245 STO 06      d
246 RCL 04
247 RCL 01
248 *
249 X↑2
250 ENTER↑
251 +
252 1
253 RCL 05
254 -
255 /
256 STO 07      U
257 RCL 02
258 RCL 03
259 *
260 X↑2
261 ENTER↑
262 +
263 RCL 05
264 /           V
265 ST+ 07      X
```

```
266 -
267 STO 08      Y
268 RCL 06
269 ENTER↑
270 SIN
271 /
272 STO 05      T
273 ENTER↑
274 +
275 X↑2
276 STO 09      D
277 RCL 06
278 COS
279 ENTER↑
280 +
281 STO 10      E
282 *
283 STO 11      A
284 RCL 10
285 -
286 2
287 /
288 RCL 05
289 -
290 CHS         C
291 RCL 07
292 *
293 RCL 11
294 +
295 RCL 07
296 *           n1
297 RCL 09
298 ENTER↑
299 +
300 STO 12      B
301 RCL 10
302 RCL 08
.303 *
```

304 +		342 RCL 07	
305 RCL 08		343 *	
306 *	n_2	344 −	
307 −		345 RCL 12	
308 RCL 09		346 4	
309 RCL 07		347 +	
310 *		348 RCL 08	
311 RCL 08		349 *	M
312 *	n_3	350 −	
313 +		351 64	
314 64		352 /	
315 /		353 RCL 14	
316 RCL 14		354 *	fM/64
317 *	$\delta_2 d/f$	355 RCL 05	
318 RCL 05		356 2	
319 RCL 07		357 /	
320 *		358 +	
321 RCL 08		359 RCL 14	
322 −		360 *	G
323 4		361 RCL 08	
324 /		362 ENTER↑	
325 −	$\delta_2 d/f - \delta_1 d/f$	363 +	
326 RCL 14		364 4	
327 *		365 RCL 07	
328 RCL 05		366 −	
329 +	$T - \delta_1 d + \delta_2 d$	367 RCL 10	
330 RCL 06		368 *	
331 SIN		369 −	F
332 *		370 *	FG
333 STO 13	S/a_e	371 RCL 00	
334 RCL 05		372 DEG	
335 32		373 TAN	
336 *		374 *	
337 RCL 05		375 4	
338 20		376 /	
339 *		377 CHS	Q
340 RCL 11		378 R-D	
341 −		379 RCL 00	

380 +		417♦LBL "MI"	MANUAL INPUT ROUTINE
381 2		418 XEQ "SW"	Swap registers, then input data
382 /		419 XEQ 15	
383 STO 09	$\Delta\lambda'_m$	420 GTO 50	
384 1		421♦LBL "CR"	CARD INPUT ROUTINE
385 P-R		422 CF 21	
386 RCL 02		423 26.035	Clear AVIEW flag
387 *		424 "1ST CAR D"	
388 X<>Y		425 AVIEW	
389 RCL 03		426 XROM 30, 03	RDTAX - wait for data card
390 *		427 16.025	
391 R-P		428 "2ND CAR D"	
392 RDN		429 AVIEW	
393 STO 12	t_1	430 XROM 30, 03	RDTAX - wait for data card
394 RCL 09		431 SF 10	Set input interlock
395 1		432 GTO 50	
396 P-R		433♦LBL 35	FIND TRIPLET VERTEX
397 RCL 01		434 SF 21	Set AVIEW flag
398 *		435 RCL 16	
399 CHS		436 RCL 26	
400 X<>Y		437 X≠Y?	ID2 = ID1 ?
401 RCL 04		438 GTO 11	
402 *		439 TONE 1	Error if "yes".
403 R-P		440 "E: ID1 = ID2"	
404 RDN	t_2	441 AVIEW	
405 ST+ 12	α_{12}	442 GTO 50	- - - - - - - - - - - - - - -
406 -		443♦LBL 11	
407 1		444 CF 05	
408 P-R		445 CF 06	
409 R-P	α_{21}	446 RCL 20	
410 RCL 12		447 RCL 30	Master 2 and Master 1 colocated?
411 X<>Y		448 -	
412 P-R		449 RCL 21	
413 R-P			
414 CLX			
415 RCL 13	Stack contains α_{21}, α_{12} and S/a_e in the Z,Y and X registers		
416 RTN			

Program	Comments
450 RCL 31	
451 -	
452 +	
453 X=0?	
454 RTN	
455 SF 05	
456 RCL 23	Slave 2
457 RCL 30	and
458 -	Master 1 Colocated?
459 RCL 24	
460 RCL 31	
461 -	
462 +	
463 X=0?	
464 RTN	
465 SF 06	
466 RCL 23	Slave 2
467 RCL 33	and
468 -	Slave 1 Colocated?
469 RCL 24	
470 RCL 34	
471 -	
472 +	
473 X=0?	
474 RTN	
475 CF 05	
476 RCL 20	Master 2
477 RCL 33	and
478 -	Slave 1 Colocated?
479 RCL 21	
480 RCL 34	
481 -	
482 +	
483 X=0?	
484 RTN	
485 TONE 1	
486 "E: NO TRIPLET"	No triplet found

Program	Comments
487 AVIEW	
488 GTO 50	
489◆LBL "PR"	PREDICT ITD ROUTINE
490 SF 01	
491 GTO 02	
492◆LBL "CA"	CALIBRATE ROUTINE
493 CF 01	
494◆LBL 02	CA and PR input routine
495 FIX 4	
496 XEQ 51	Flattening Factor
497 SF 21	Set AVIEW flag
498 SF 14	Set loop control
499 ADV	
500 "LAT? "	Input
501 XEQ 99	latitude
502 XEQ 14	
503 STO 37	
504 "LON? "	Input
505 XEQ 99	longitude
506 STO 38	
507◆LBL 66	Begin loop
508 FS? 01	Transfer if PR input
509 GTO 02	
510 ": "	
511 ASTO 00	
512 BEEP	Input benchmark
513 "ITD "	
514 ARCL 26	ITD.
515 ARCL 00	
516 XEQ 99	
517 STO 39	Initialize CA calculation
518 CLX	
519 GTO 08	Transfer
520◆LBL 02	
521 CLX	Initialize PR calculation
522 STO 39	
523◆LBL 08	CA and PR computations
524 RCL 37	

525 RCL 38 526 RCL 33 527 RCL 34	Load stack.
528 XEQ 80 529 XEQ 03 530 ST- 39	Reverse solution from Slave. Secondary phase corr.
531 RCL 37 532 RCL 38 533 RCL 30 534 RCL 31	Load stack.
535 XEQ 80 536 XEQ 03 537 RCL 39 538 +	Reverse solution from Master Secondary phase correction.
539 FS? 01 540 GTO 02	Branch for PR
541 RCL 29 542 - 543 STO 27 544 GTO 08	Correct baseline to benchmark
545◆LBL 02 546 CHS 547 RCL 27 548 + 549 RCL 29 550 + 551 ": " 552 ASTO 00 553 CLA 554 ARCL 26 555 ARCL 00 556 FIX 2 557 ARCL X 558 BEEP 559 AVIEW	Compute predicted ITD. Display predicted ITD
560◆LBL 08 561 XEQ "SW" 562 FS?C 14	Swap station data

563 GTO 66	Loop for second station
564 CF 01 565 GTO 50	Done
566◆LBL "CS" 567 16.025 568 CF 21 569 "WRITE: " 570 ARCL 16 571 AVIEW	CARD STORE ROUTINE Prompt with station ITD
572 XROM 30,08	WDATX
573◆LBL 50 574 CF 21 575 TONE 6 576 "NEXT OPTION?" 577 AVIEW 578 STOP 579 GTO 50	"NEXT OPTION?" Query
580◆LBL 51 581 DEG 582 298.26 583 1/X 584 STO 14 585 RTN	STORE FLATTENING FACTOR
586◆LBL 44 587◆LBL 14 588 TAN 589 1 590 RCL 14 591 - 592 * 593 ATAN 594 RTN	CONVERT GEOGRAPHIC LATITUDE TO PARAMETRIC LATITUDE
595◆LBL 15 596 ADV 597 FIX 0	STATION DATA INPUT PROMPTS

598 SF 21	
599 CF 23	Clear alpha entry flag
600 "ID? "	
601 AON	
602 PROMPT	Input station ID
603 ASTO X	
604 CLA	
605 AOFF	
606 FC? 23	
607 RTN	
608 SF 09	
609 XEQ 01	Input
610 STO 16	Coding
611 "CODE DE LAY? "	delay
612 XEQ 00	
613 STO 19	
614 FIX 6	Input master
615 "MS LAT? "	station latitude.
616 XEQ 00	
617 XEQ 14	Convert to Parametric.
618 STO 20	
619 "MS LON? "	Input master
620 XEQ 00	station longitude.
621 STO 21	
622 "SS LAT? "	Input Slave
623 XEQ 00	station latitude.
624 XEQ 14	Convert to parametric.
625 STO 23	
626 "SS LON? "	Input Slave
627 XEQ 00	station longitude.
628 STO 24	
629 RCL 20	
630 RDN	

631 XEQ 80	Compute reverse solution to
632 STO 18	determine baseline and
633 RDN	master-slave forward and
634 STO 22	reverse azimuths.
635 RDN	
636 STO 25	
637 RCL 18	Add secondary phase correc-
638 XEQ 03	tion to baseline.
639 STO 17	
640 RTN	
641♦LBL 99	DATA INPUT PROMPT AND
642♦LBL 00	PRINT
643 PROMPT	
644♦LBL 01	
645 ARCL X	
646 SF 25	Set error ignore flag
647 FS? 55	Print
648 PRA	
649 CF 25	Clear error ignore flag
650 FC?C 09	Convert DMS to DEG if
651 HR	FØ9 clear.
652 RTN	
653♦LBL 03	SECONDARY PHASE CORRECTION
654 21282.36	$a_e \eta / V_o$
655 *	
656 ENTER↑	
657 ENTER↑	
658 537	
659 X>Y?	
660 GTO 04	- - - - - - - - - - - - - -
661 CLX	
662 129	
663 X<>Y	$T \geq 537$ µS
664 /	
665 LASTX	
666 6458 E-7	
667 *	
668 +	

Program	Comments
669 .408	
670 -	
671 +	
672 RTN	
673♦LBL 04	
674 CLX	
675 2.74	T < 537 µS
676 X<>Y	
677 /	
678 LASTX	
679 33 E-5	
680 *	
681 +	
682 .011	
683 -	
684 +	
685 RTN	
686♦LBL 05	TIME DELAY VALIDITY CHECK
687 CF 00	
688 LASTX	
689 X<=Y?	
690 GTO 06	
691 CHS	Okay if
692 X>Y?	
693 GTO 06	$\lvert TD \rvert < T_B + p(T_B)$
694 CLX	
695 21295.87	a_p
696 /	
697 RTN	2a in X-register
698♦LBL 06	
699 SF 21	ITD input error
700 SF 00	
701 TONE 1	Set FØØ to requery.
702 "E: ITD ERROR"	
703 AVIEW	
704 RTN	

Program	Comments
705♦LBL "FI"	FIXING ALGORITHM
706 FS?C 10	Find vertex if interloc flag 1Ø is set.
707 XEQ 35	
708 SF 21	
709 ADV	
710♦LBL 10	
711 XEQ 51	Generate flattening
712 SF 09	No DMS conversion
713 FIX 2	
714 " "	
715 ASTO 01	Input ITD of 1st pair
716 "ITD: "	
717 ASTO 00	
718 ARCL 26	
719 ARCL 01	
720 XEQ 00	
721 RCL 29	
722 -	Compute TD
723 RCL 27	
724 -	
725 XEQ 05	TD validity check
726 FS?C 00	
727 GTO 10	Requery ITD
728 STO 36	Store $2a_1$
729♦LBL 07	
730 SF 09	No DMS conversion
731 CLA	
732 ARCL 00	Input ITD of 2nd pair
733 ARCL 16	
734 ARCL 01	
735 XEQ 00	
736 RCL 19	
737 -	Compute TD
738 RCL 17	
739 -	
740 XEQ 05	TD validity check
741 FS?C 00	
742 GTO 07	Requery ITD

743 STO 15	Store $2a_2$
744♦LBL 18	ALTERNATE SOLUTION entry.
745 FS? 10	Input ITD's if vertex
746 GTO "FI"	interlock set.
747 XEQ 51	Regenerate flattening
748 RAD	
749 RCL 15	
750 1	
751 P-R	
752 X<>Y	
753 FS? 05	Station 2 vertex flag
754 CHS	
755 STO 01	A_2
756 CLX	
757 RCL 18	
758 1	
759 P-R	
760 X<>Y	
761 STO 03	C_2
762 RDN	
763 -	
764 STO 02	B_2
765 RCL 36	
766 1	
767 P-R	
768 X<>Y	
769 FS? 06	Station 1 vertex flag
770 CHS	
771 STO 04	A_1
772 CLX	
773 RCL 28	
774 1	
775 P-R	
776 X<>Y	
777 STO 06	C_1
778 RDN	
779 -	
780 STO 05	B_1

781 RCL 01	
782 *	
783 RCL 02	
784 RCL 04	K
785 *	
786 -	
787 STO 07	
788 DEG	
789 RCL 22	
790 FS? 05	Station 2 vertex flag
791 RCL 25	
792 STO 11	
793 RCL 05	
794 RCL 03	
795 *	
796 P-R	
797 STO 08	$B_1C_2 \cos \xi_2$
798 RDN	
799 STO 09	$B_1C_2 \sin \xi_2$
800 RCL 32	
801 FS? 06	Station 1 vertex flag
802 RCL 35	
803 RCL 02	
804 RCL 06	
805 *	
806 P-R	
807 RCL 08	
808 -	C
809 X<>Y	
810 RCL 09	
811 -	S
812 X<>Y	
813 R-P	ρ and γ
814 RCL 07	
815 X<>Y	
816 /	
817 ACOS	$\cos^{-1} (K/\rho)$
818 FS? 03	Alternate solution?

819 CHS	
820 +	
821 STO 10	$\alpha = \gamma \pm \cos^{-1}(K/\rho)$
822 RCL 11	
823 -	
824 COS	
825 RCL 03	
826 *	
827 RCL 01	
828 +	$C_2 \cos(\alpha-\xi_2) + A_2$
829 RCL 02	
830 X<>Y	
831 R-P	
832 RDN	
833 D-R	
834 STO 12	r
835 RCL 20	
836 RCL 21	
837 FC? 05	Station 2 vertex flag
838 GTO 09	
839 RCL 23	
840 RCL 24	
	- - - - - - - - - - - - - -
841♦LBL 09	
842 RCL 10	
843 RCL 12	
844 XEQ 90	Compute direct solution
845 STO 37	
846 FIX 4	θ_{fix}
847 XEQ 19	
848 SF 21	θ_{fix}
849 "LAT: "	
850 ARCL X	Display latitude
851 BEEP	
852 AVIEW	
853 RDN	
854 STO 38	λ_{fix}
855 XEQ 20	
856 "LON: "	

857 ARCL X	
858 AVIEW	Display longitude
859 GTO 50	
860♦LBL 19	CONVERT
861 TAN	PARAMETRIC LATITUDE
862 1	TO
863 RCL 14	GEOGRAPHIC LATITUDE
864 -	
865 /	
866 ATAN	
867♦LBL 20	DMS ROUNDING ROUTINE
868 HMS	
869 RND	
870 0	
871 HMS+	
872 RTN	
873♦LBL "DH"	DISTANCE AND HEADING
874 SF 21	Any origin to
875 ADV	
876 XEQ 51	any destination
877 "ORIG LAT? "	Input origin latitude
878 XEQ 99	
879 XEQ 44	Convert to parametric
880 STO 37	
881 "ORIG LON? "	Input origin longitude
882 XEQ 99	
883 STO 38	
884 SF 08	Set FØ8 to compute
885♦LBL "DN"	STORE DESTINATION
886 SF 21	
887 FC? 08	
888 ADV	
889 XEQ 51	
890 "DEST LAT? "	Input destination latitude
891 XEQ 99	

Program	Comments
892 XEQ 44	Convert to parametric
893 STO 40	
894 "DEST LON? "	Input destination longitude
895 XEQ 99	
896 STO 41	
897 FC?C 08	Return if FØ8 not set to compute.
898 GTO 50	
899♦LBL "HD"	COMPUTE DISTANCE AND HEADING
900 RCL 37	
901 RCL 38	
902 RCL 40	
903 RCL 41	
904 XEQ 80	Compute reverse solution
905 3443.917	Convert distance to n. mi.
906 *	
907 BEEP	
908 SF 21	
909 "N.MI: "	Display distance
910 FIX 2	
911 ARCL X	
912 AVIEW	
913 RDN	
914 FIX 4	
915 360	$0° \leq$ bearing $< 360°$
916 MOD	
917 STO 39	
918 XEQ 20	Round DMS
919 "BRG: "	
920 ARCL X	Display bearing
921 AVIEW	
922 GTO 50	
923♦LBL "AS"	COMPUTE ALTERNATE SOLUTION
924 FS?C 03	
925 GTO 18	
926 SF 03	Toggle FØ3
927 GTO 18	
928♦LBL "ED"	ECHO DATA ROUTINE
929 SF 00	
930 XEQ "SW"	
931 XEQ 13	
932 XEQ "SW"	
933 XEQ 13	
934♦LBL "CE"	CARD ECHO ROUTINE
935 CF 21	
936 16.025	
937 "STA. CARD"	
938 AVIEW	
939 XROM 30,03	RDTAX
940♦LBL 13	Data review of R16-R25
941 FIX 0	
942 XEQ 51	Regenerate flattening
943 SF 21	
944 ADV	
945 "ID: "	Display station ID
946 ARCL 16	
947 AVIEW	
948 "CD: "	Display coding delay
949 ARCL 19	
950 AVIEW	
951 "MLT: "	Display master station latitude
952 RCL 20	
953 FIX 6	
954 XEQ 19	
955 ARCL X	
956 AVIEW	
957 "MLN: "	Display master station longitude
958 RCL 21	
959 XEQ 20	
960 ARCL X	
961 AVIEW	
962 "SLT: "	
963 RCL 23	

```
964 XEQ 19           Display slave station
965 ARCL X               latitude
966 AVIEW
967 "SLN: "
968 RCL 24           Display slave station
969 XEQ 20               longitude
970 ARCL X
971 AVIEW
972 FIX 3            Display base line
973 "BL: "               in microseconds
974 ARCL 17
975 AVIEW
976 FS?C 00
977 RTN
978 GTO 50
979♦LBL "SW"         SWAP STATION DATA
980 SF 10                Swaps R16-R25
981 35.025               with R26-R35
982 STO 01
983 25
984 STO 02
985♦LBL 12           Begin loop
986 RCL IND
02
987 X<> IND
01
988 STO IND
02
989 DSE 02
990 " "
991 DSE 01
992 GTO 12           Loop to LBL 12
993 RTN
994♦LBL "XR"         XMEM RECALL
995 SF 07
996 GTO 02
997♦LBL "TR"         TAPE RECALL
998 CF 07
```

```
999♦LBL 02
1000 XEQ "SW         Swap data
"
1001 XEQ 60          ID Prompt
1002 ASTO 16         Store ID
1003 XEQ 08          Set pointer
1004 FS? 07
1005 XROM 25         GETRX - Input from XMEM
,21
1006 FC? 07
1007 XROM 28         READRX - Input from tape
,09
1008 GTO 50
1009♦LBL "XS         XMEM STORE
"
1010 SF 07
1011 GTO 02
1012♦LBL "TS         TAPE STORE
"
1013 CF 07
1014♦LBL 02
1015 CLA
1016 ARCL 16         Recall ID
1017 9
1018 FS? 07
1019 XROM 25         CRFLD - XMEM create
,11
1020 FC? 07
1021 XROM 28         CREATE - Tape create
,01
1022 XEQ 08          Set pointer & register
1023 FS? 07                    range
1024 XROM 25         SAVERX-Save in XMEM
,40
1025 FC? 07
1026 XROM 28         WRTRX - Save on tape
,22
1027 GTO 50
```

1028◆LBL "XD"	XMEM DELETE
1029 SF 07 1030 GTO 02	
1031◆LBL "TD"	TAPE DELETE
1032 CF 07 1033◆LBL 02 1034 XEQ 60 1035 FS? 07	- - - - - - - - - - - - - -
1036 XROM 25,31	PURFL - Purge from XMEM
1037 FC? 07 1038 XROM 28,04	PURGE - Purge from tape
1039 GTO 50	
1040◆LBL 60 1041 CF 21 1042 "ID?" 1043 AON 1044 PROMPT 1045 AOFF 1046 RTN	ID INPUT PROMPT
1047◆LBL 08 1048 0 1049 FS? 07	SET POINTER
1050 XROM 25,43	SEEKPT - Set XMEM pointer to zero
1051 FC? 07 1052 XROM 28,14	SEEKR - Set tape pointer to zero
1053 17.025 1054 .END.	R17-R25 to be transmitted.

Appendix B: STATION COVERAGE

Station	No. of Pairs	Location
4990	2	Central Pacific
5930	2	East Coast, Canada
5990	3	West Coast, Canada
7930	3	North Atlantic
7960	2	Gulf of Alaska
7970	4	Norwegian Sea
7980	4	Southeast U.S.A.
7990	3	Mediterranean Sea
8970	3	Great Lakes
9940	3	West coast, U.S.A.
9960	4	Northeast U.S.A.
9970	4	Northwest Pacific
9990	3	North Pacific

Appendix C: LORAN-C STATION DATA

The following list contains the pertinent parameters for each Loran-C station pair. This list was compiled from the data in Reference 4. Each column contains the following information:

1. The Loran-C station pair designator.
2. The coding delay.
3. The master station latitude.
4. The master station longitude.
5. The slave station latitude.
6. The slave station longitude.

In this list, positive longitudes are West, negative longitudes are East, positive latitudes are North and negative latitudes are South. In columns 3 through 6 the latitudes and longitudes appear to be in decimal form, but the actual format is DDD.MMSSFF (which is compatible with the HP-41CV D.MS or H.MS input mode) where

DDD designates degrees,
MM designates minutes,
SS designates seconds, and
FF designates hundredths of seconds.

ID	CD	MS LAT	MS LON	SS LAT	SS LON
4990X	11000	[illegible]	[illegible]	[illegible]	155.530970
4990Y	29000	[illegible]	[illegible]	[illegible]	178.173020
5930X	11000	[illegible]	[illegible]	[illegible]	069.583909
5930Y	25000	[illegible]	[illegible]	[illegible]	053.102816
5990X	11000	[illegible]	[illegible]	[illegible]	131.151965
5990Y	27000	[illegible]	[illegible]	[illegible]	119.443953
5990Z	41000	[illegible]	[illegible]	[illegible]	127.212935
7930W	11000	[illegible]	[illegible]	[illegible]	023.552175
7930X	21000	[illegible]	[illegible]	[illegible]	007.042671
7930Z	43000	[illegible]	[illegible]	[illegible]	053.102816
7960X	11000	[illegible]	[illegible]	[illegible]	152.221122
7960Y	26000	[illegible]	[illegible]	[illegible]	131.151965
7970W	36000	[illegible]	[illegible]	[illegible]	-008.173633
7970X	11000	[illegible]	[illegible]	[illegible]	-014.274700
7970Y	46000	[illegible]	[illegible]	[illegible]	+023.552175
7970Z	60000	[illegible]	[illegible]	[illegible]	+008.435869
7980W	11000	[illegible]	[illegible]	[illegible]	090.494360
7980X	23000	[illegible]	[illegible]	[illegible]	097.500009
7980Y	43000	[illegible]	[illegible]	[illegible]	080.065352
7980Z	59000	[illegible]	[illegible]	[illegible]	077.544676
7990X	11000	[illegible]	[illegible]	[illegible]	-012.312996
7990Y	29000	[illegible]	[illegible]	[illegible]	-027.520152
7990Z	47000	[illegible]	[illegible]	[illegible]	-003.121590
8970W	11000	[illegible]	[illegible]	[illegible]	085.100930
8970X	28000	[illegible]	[illegible]	[illegible]	076.493386
8970Y	44000	[illegible]	[illegible]	[illegible]	094.331847
9940W	11000	[illegible]	[illegible]	[illegible]	119.443953
9940X	27000	[illegible]	[illegible]	[illegible]	122.294453
9940Y	40000	[illegible]	[illegible]	[illegible]	114.481743
9960W	11000	[illegible]	[illegible]	[illegible]	067.553771
9960X	25000	[illegible]	[illegible]	[illegible]	069.583909
9960Y	39000	[illegible]	[illegible]	[illegible]	077.544676
9960Z	54000	[illegible]	[illegible]	[illegible]	087.291214
9970W	11000	[illegible]	[illegible]	[illegible]	-153.58515
9970X	30000	[illegible]	[illegible]	[illegible]	-143.430906
9970Y	55000	[illegible]	[illegible]	[illegible]	-128.085621
9970Z	75000	[illegible]	[illegible]	[illegible]	-138.095523
9990X	11000	[illegible]	[illegible]	[illegible]	+173.105231
9990Y	29000	[illegible]	[illegible]	[illegible]	+166.531447
9990Z	43000	[illegible]	[illegible]	[illegible]	+152.221122

Appendix D: COLOCATED STATIONS

5930 MASTER	- 9960W
5930X	- 9960X
5930Y	- 7930Z
5990X	- 7960Y
5990Y	- 9940W
7930W	- 7970Y
7930X	- 7970 MASTER
7960X	- 9990Z
7980Z	- 9960Y
7980 MASTER	- 8970W
8970X	- 9960 MASTER
8970 MASTER	- 9960Z

Slave stations are denoted with a letter suffix. Master stations are so designated.

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